Hobby-Mathematiker

An: Susanne Albers
Kiehlufer 125-129
12059 Berlin

- Lösung der Proportionsstudie von Leonardo da Vinci, seiner Zeichnung mit dem Vitruvianischen Menschen. **Formel:** $D_{Kreis} = L_{Quadrat} * [1 – 0{,}5*\sqrt{(4/\pi)}] / \sqrt{[\sqrt{(4/\pi)} – 1]} = L_{Quadrat} * 1{,}21632695..$

Sehr geehrte Frau Albers,

mit diesem Brief erhalten Sie beiliegend die Lösung der Proportionsstudie von Leonardo da Vinci, seiner Zeichnung mit dem Vitruvianischen Menschen.

Der Kreisdurchmesser auf der Zeichnung mit dem Vitruvianischen Menschen von L. da Vinci lässt sich durch Gleichsetzung von zwei Geraden-Funktionen und unter Betrachtung von zwei Quadrat-/Kreis-Paaren auf relativ einfache Weise berechnen. Den Paaren liegt jeweils Flächengleichheit zugrunde. Damit lässt sich eine mathematische Beziehung zwischen der Quadratseitenlänge (L_{Q1}) und dem Kreisdurchmesser (in der Folge mit D_2 angegeben) auf der Proportionsstudie aufstellen.

Wenn Sie sich die Skizze 1 des beiliegenden Kurzberichtes anschauen, ist dies leicht zu erkennen. Ein Kreisdurchmesser ist - wie im Falle der Proportionsstudie - unter folgenden Bedingungen definiert:

1) der Kreis liegt unten tangential an einer Linie (z.B. eine untere Linie eines Quadrates) an.
2) die vertikale Symmetrieachse, auf der der Kreismittelpunkt liegt, ist bekannt (Diese Achse kann die vertikale Symmetrieachse eines Quadrates sein).
3) ein Punkt auf der Kreisbahn, der nicht auf der vertikalen Symmetrieachse liegt, ist definiert.

Bis jetzt – soweit mir bekannt ist – ist die Lösung für Bedingung 3) bezüglich der Proportionsstudie noch nicht gefunden worden.

Lösungsansatz: der unter 3) gesuchte Punkt ist der Schnittpunkt von den beiden Geraden-Funktionen $g_1(x)$ und $g_2(x)$ (siehe nochmals Skizze 1).

In dem Fall der Proportionsstudie geht man gemäß dem Buch „Ich aber quadriere den Kreis“ von zwei Quadrat/Kreis-Paaren aus. Für die Durchmesser D und Quadratseitenlängen L_Q liegen bei Flächengleichheit folgende Beziehungen vor:

$D_1 = L_{Q1} * \sqrt{(4/\pi)}$ für Quadrat/Kreis-Paar 1 und $D_2 = L_{Q2} * \sqrt{(4/\pi)}$ für Quadrat/Kreis-Paar 2

Damit ist eine Beziehung zwischen der Quadratseitenlänge L_{Q1} und dem Kreisdurchmesser D_2 bei der geometrischen Konstellation (der Skizze 1), die der auf der Zeichnung von Leonardo da Vinci entspricht, mittels Gleichsetzung der Geraden-Funktionen $g_1(x)$ und $g_2(x)$ herstellbar.

Der Durchmesser D_2 auf der Zeichnung von L. da Vinci beträgt nach der Kreiszahl π aufgelöst:

$D_2 = L_{Q1} * [1 – 0{,}5*\sqrt{(4/\pi)}] / \sqrt{[\sqrt{(4/\pi)} – 1]} = 12{,}1632695..$ mit Quadratseitenlänge $L_{Q1}=10$

Die Herleitung dieser Gleichung erschließt sich aus den beiliegenden Berichten.

Und siehe da, der mit dieser Vorgehensweise berechnete Durchmesser passt sehr genau - mit einer relativen Abweichung von 0,05% - zu dem Durchmesser-Messwert, der laut Buch “Ich aber quadriere den Kreis“ ~~[1]~~ von K. Schröer/K. Irle aus der Zeichnung von Leonardo da Vinci gewonnen wurde.

Ist das Zufall? Wohl kaum, zumal zwei Mal die Quadratur des Kreises bezüglich der Flächengleichheit von Quadrat zu Kreis bei der mathematischen Beziehung einfließt.
In dem Buch “Ich aber quadriere den Kreis“ von K. Schröer/K. Irle sind einige Zitate von Leonardo da Vinci angegeben, wo er von der Quadratur des Kreises spricht und unter anderem den Punkt als Grundprinzip der Geometrie nennt.

siehe Rückseite

Den zwei Seiten umfassenden Kurzbericht und den längeren 7 Seiten umfassenden Bericht in Englisch oder darauf Aufbauendes können Sie gerne veröffentlichen, falls Sie der Meinung sind, dass diese Ausführungen von Relevanz sind.

Ich danke Ihnen schon vielmals für das von Ihnen entgegengebrachte Interesse und wünsche Ihnen Alles Gute.

Mit freundlichen Grüßen, den 09.11.2020

Hobby-Mathematiker

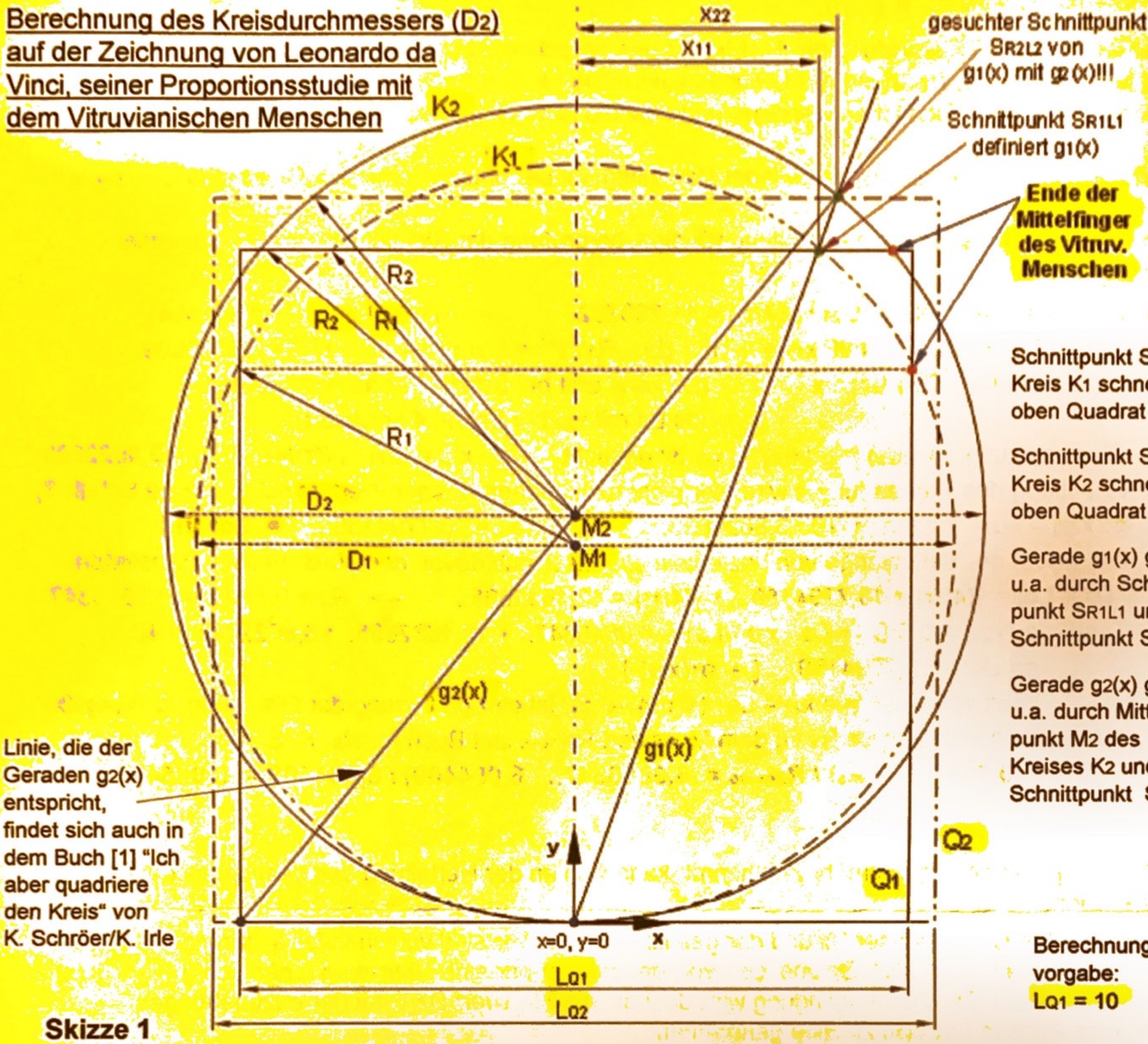
PS:
(Auszug der Seite **4**)
<u>Questions referring to the measures of the proportion study:</u>

The diameter at da Vinci´s drawing of the proportion study is **220 mm** according to a report from 2015 of Vitor Murtinho. The square length is measured to 181,5 mm according to this report.

My question especially to art scientists is: 500 years ago, in this era, was the metric system or a length system similar to it already known?

Since the antique era the approximation **"22 : 7"** has been known for the circular figure π.
Can it be that there is a connection between the measure 220 mm of the diameter at da Vinci´s drawing and the figure 22 in the approximation formula for the circular figure?

Berechnung des Kreisdurchmessers (D_2) auf der Zeichnung von Leonardo da Vinci, seiner Proportionsstudie mit dem Vitruvianischen Menschen



Schnittpunkt S_{R1L1}: Kreis K_1 schneidet oben Quadrat Q_1

Schnittpunkt S_{R2L2}: Kreis K_2 schneidet oben Quadrat Q_2

Gerade $g_1(x)$ geht u.a. durch Schnittpunkt S_{R1L1} und Schnittpunkt S_{R2L2}

Gerade $g_2(x)$ geht u.a. durch Mittelpunkt M_2 des Kreises K_2 und Schnittpunkt S_{R2L2}

Berechnungsvorgabe: $L_{Q1} = 10$

Skizze 1

Das Quadrat Q_1 mit der Seitenlänge L_{Q1} und der Kreis K_1 mit dem Durchmesser D_1 sind flächengleich[1*]. Das Quadrat Q_1 umgibt den Vitruvianischen Menschen, der Kreis K_1 berührt die Mittelfinger der waagrecht ausgestreckten Arme. 1*) Fläche $F_{Q1} = L_{Q1} \cdot L_{Q1}$ = Fläche $F_{K1} = D_1 \cdot D_1 \cdot \pi / 4$

Das Quadrat Q_2 mit der Seitenlänge L_{Q2} und der Kreis K_2 mit dem Durchmesser D_2 sind ebenfalls flächengleich[2*]. Der Kreis K_2 berührt die Mittelfinger der schräg nach oben ausgestreckten Arme. **2*)** Fläche $F_{Q2} = L_{Q2} \cdot L_{Q2}$ = Fläche $F_{K2} = D_2 \cdot D_2 \cdot \pi / 4$ (1* und 2* nach [1]) ---> $D_2 = L_{Q2} * \sqrt{4/\pi}$

Der gestrichelte Kreis K_1 und das gestrichelte Quadrat Q_2 auf Skizze 1 sind nicht auf der Zeichnung von L. da Vinci dargestellt. Auf Skizze 1 fehlt der Vitruvianische Mensch. Positionen der Mittelfinger-Enden sind angezeigt. Mittelpunkt M_2 stimmt mit dem Nabel-Punkt des Vitruvian. Menschen überein.

$g_1(x) = L_{Q1} / x_{11} \cdot x$ mit $x_{11} = [R_1^2 - (L_{Q1}-R_1)^2]^{0,5} = 0{,}3583004 \cdot L_{Q1}$ und $R_1 = 0{,}5 \cdot D_1 = 5{,}641896..$
$g_2(x) = 0{,}5 \cdot D_2 + D_2 / L_{Q1} \cdot x$; **Durchmesser D_2 ist gesucht** | aus $D_1 = L_{Q1} \cdot \sqrt{4/\pi} = 11{,}283792..$

Gleichsetzung "**$g_1(x) = g_2(x)$**" der beiden Geraden-Funktionen führt zu dem gesuchten x-Wert x_{22}:

$x_{gesucht} = \mathbf{x_{22} = L_{Q1} \cdot \sqrt{\pi/4} - 0{,}5 \cdot L_{Q1}} = 3{,}8622693..$ mit Quadrat-Seitenlänge **$L_{Q1}=10$**

$L_{Q2} = g_1(x_{22}) = x_{22} \cdot L_{Q1} / x_{11} = \mathbf{L_{Q1} \cdot [\sqrt{\pi/4} - 0{,}5] / [\sqrt{4/\pi} - 1]^{0,5}} = 10{,}7794169..$ **$[= g_2(x_{22})]$**

$D_2 = L_{Q2} \cdot \sqrt{4/\pi} = \mathbf{L_{Q1} \cdot [1 - 0{,}5 \cdot \sqrt{4/\pi}] / [\sqrt{4/\pi} - 1]^{0,5}} = 12{,}1632695..$ $[= 2 \cdot R_2]$

Abweichung vom berechneten Radius R_2 zu dem gemessenen Radius "$R_{2mess} = 6{,}084409$" laut dem Buch [1] bezogen auf eine Quadrat-Seitenlänge "$L_{Q1}=10$" ist:

$\Delta R_2 = (0{,}5 \cdot D_2 - R_{2mess}) / R_{2mess} = (6{,}0816347 - 6{,}084409) / 6{,}084409 = -0{,}0454\%$

Für die folgende Überprüfung der Richtigkeit des berechneten Durchmessers D_2 benötigt man etwa 30 Minuten. Die Überprüfung der Lösungs-Herleitung für den Armradius sowie für den Ansatz einer möglichen Beinstellung ist aufwendiger und deshalb nicht Gegenstand dieses Kurzberichtes.

Wichtig: das Koordinatenkreuz befindet sich mittig auf der unteren Linie des Quadrates Q_1 bzw. Q_2.

Hier sei für Interessierte eine Anleitung für die Überprüfung der Richtigkeit vom Durchmesser D_2 angegeben:

1) $L_{Q1} = 10$ ---> $D_1 = L_{Q1} \cdot \sqrt{(4/\pi)} = 11{,}283792..$ ---> $R_1 = 0{,}5 \cdot D_1 = 5{,}641896..$
2) $g_1(x) = L_{Q1} \cdot x / x_{11}$ mit $x_{11} = [R_1^2 - (L_{Q1}-R_1)^2]^{0,5} = L_{Q1} \cdot [\sqrt{(4/\pi)} - 1]^{0,5} = 3{,}5830039..$
3) $g_2(x) = 0{,}5 \cdot D_2 + D_2 / L_{Q1} \cdot x$ (D_2 kann erst bei Schritt 6 berechnet werden, da D_2 mit x_{22} bei Schritt 4 korreliert)
4) Gleichsetzung: $g_1(x_{22}) = g_2(x_{22})$; x_{22} ist gesucht ---> $\mathbf{x_{22} = L_{Q1} \cdot (\sqrt{(\pi/4)} - 0{,}5) = 3{,}8622693..}$
   Ermittlung des Wertes für x_{22} wird hier nicht beschrieben. Dieser Wert ist richtig (siehe Schritt 7).
5) $g_1(x_{22}) = L_{Q1} \cdot x_{22} / x_{11} = 10 \cdot 3{,}8622693.. / 3{,}5830039.. = 10{,}7794169..$ **[= L_{Q2} !!!]**
6) Da $g_1(x_{22})$ der Seitenlänge von L_{Q2} entspricht, lässt sich davon der Durchmesser D_2 ableiten.
   $\mathbf{D_2 = L_{Q2} \cdot \sqrt{(4/\pi)} = 10{,}7794169.. \cdot \sqrt{(4/\pi)} = 12{,}1632695..}$ ---> $R_2 = 0{,}5 \cdot D_2 = 6{,}0816347..$
7) Test: $g_2(x_{22}) = 0{,}5 \cdot D_2 + D_2 \cdot x_{22} / L_{Q1} = 6{,}0816347.. + 12{,}1632695.. \cdot 3{,}8622693.. / 10$
   $g_2(x_{22}) = 10{,}7794169..$ **[= $g_1(x_{22})$]**
   Wert von x mit "$x = x_{22} = L_{Q1} \cdot (\sqrt{(\pi/4)} - 0{,}5)$" ist einzige Lösung der Gleichung "$g_1(x)=g_2(x)$".
8) Abweichung vom Radius R_2 zu dem Messwert R_{2mess} laut Buch [1] von K. Schröer / K. Irle:
   $\mathbf{\Delta R_2 = (0{,}5 \cdot D_2 - R_{2mess}) / R_{2mess} = (6{,}0816347.. - 6{,}084409) / 6{,}084409 = -0{,}0454\%}$

Wer sich jetzt noch etwas mehr Zeit nimmt, kann sich an der Herleitung des gesuchten x-Wertes x_{22} versuchen:

Der gesuchte Durchmesser D_2 und der gesuchte x-Wert x_{22} sind voneinander abhängig. Es liegen zwei Unbekannte - nämlich D_2 und x_{22} - vor, denen aber nur eine Gleichung – nämlich $g_1(x) = g_2(x)$ - gegenübersteht. Zur Lösungsfindung wird deshalb ein Startwert D_{2_0} mit der dazugehörigen Geraden-Funktion $g_{2\_0}(x)$ zu Hilfe genommen.

Man kann einen beliebigen Startdurchmesser D_{2_0} auswählen. Die Terme unter der Wurzel sollten dabei aber positive Werte annehmen.
Am besten rechnet man mit dem Startdurchmesser $D_{2\_0} = L_{Q1}$, da sich damit die Startgerade $g_{2\_0}(x)$ wesentlich vereinfachen lässt.

1*) $D_{2\_0} = L_{Q1} = 10$ ---> $R_{2\_0} = 0{,}5 \cdot L_{Q1} = 5$ ---> $L_{Q2\_0} = L_{Q1} \cdot \sqrt{(\pi/4)} = 8{,}8622693..$
2*) $g_{2\_0}(x) = 0{,}5 \cdot D_{2\_0} + D_{2\_0} / L_{Q1} \cdot x = 0{,}5 \cdot L_{Q1} + x$
3*) der gesuchte x-Wert x_{22} unterliegt der Gleichung "$g_{2\_0}(x_{22}) = L_{Q2\_0}$" --->
   $g_{2\_0}(x_{22}) = 0{,}5 \cdot L_{Q1} + x_{22} = L_{Q2\_0} = L_{Q1} \cdot \sqrt{(\pi/4)}$ ---> $\mathbf{x_{22} = L_{Q1} \cdot (\sqrt{(\pi/4)} - 0{,}5) = 3{,}8622693..}$
   Warum die Lösung des gesuchten x-Wertes x_{22} der Gleichung "$g_{2\_0}(x_{22}) = L_{Q2\_0}$" unterliegt, weiß ich auch nicht. Profi-Mathematiker werden dafür sicher einen Beweis liefern können.
4*) Bei diesem Schritt hier geht es weiter wie oben beschrieben bei den Schritten 5) bis 8)
5*) Die Beinstellung könnte auf die Gerade zurückgeführt werden, die durch den Mittelpunkt M_2 des Kreises K_2 und den Schnittpunkt "obere Linie des Quadrates Q_2 mit Kreis K_1" geht.

Mag sein, dass es ein Lösungsverfahren für die gesuchten Werte von D_2 und x_{22} gibt, bei dem kein Startwert D_{2_0} notwendig ist. Nichtsdestotrotz liefert das hier vorgestellte Verfahren eine plausible und nachvollziehbare Lösung für den Durchmesser auf der Zeichnung von Leonardo da Vinci.
Dies wird durch die sehr geringe Abweichung zu dem Messwert laut dem Buch [1] untermauert.
Und daraus kann man schließen, dass Leonardo da Vinci sehr genau die Geometrien mit den Quadraturen des Kreises bezüglich der Flächengleichheit auf seiner Proportionsstudie wiedergab.

Vitruvian man – Determination of the circle diameter at the drawing of Leonardo da Vinci

In my free time I am engaged by general mathematics. Since some time I try to duplicate the geometry of the proportion study of Leonardo da Vinci.
According to my calculation the diameter D_2 at the drawing of Leonardo da Vinci amounts to:

$$D_2 = L_{Q1} \cdot [1 - 0{,}5 \cdot \sqrt{(4/\pi)}] \,/\, [\sqrt{(4/\pi)} - 1]^{0{,}5} = 12{,}1632695.. \quad \text{with square length } L_{Q1} = 10$$

A great help in this matter was the german book "Ich aber quadriere den Kreis [But me, I square the circle]" of the german authors K. Schröer / K. Irle [Statement of this book is done in the following by the shortcut [1], the figure "1" in brackets.

Besides the measured values taken from the drawing, important informations of the book [1] in my opinion are:
1) the square Q_1 with the length L_{Q1} and the circle C_1 with the diameter D_1 are equal in surface. The square Q_1 surrounds the Vitruvian man, the circle C_1 touches the middle finger of the horizontal outstreched arms. Circle C_1 is not shown at the drawing of Leonardo da Vinci.
2)The square Q_2 with the length L_{Q2} and the circle C_2 with the diameter D_2 are also equal in surface. The circle C_2 touches the middle finger of the oblique outstreched arms. Square L_{Q2} is not shown at the drawing of Leonardo da Vinci.
3) figure M9 on page 125 of [1] shows a line, which is equivalent to straight line $g_2(x)$ at sketch 1 in this report. Straight line $g_1(x)$ was known before, because I drafted it at one of my first sketches. That inspired me to get the solution by the equation of the straight lines $g_1(x)$ and $g_2(x)$.

The value for diameter D_2 is obtained by the equation of two straight lines named $g_1(x)$ and $g_2(x)$ (see sketch 1 on page 5). The aim is to get the value x which fulfills the equation:

$g_1(x) = g_2(x)$

$g_1(x) = L_{Q1} / c_{R1L1} \cdot x$ with $c_{R1L1} = [R_1^2 - (L_{Q1}-R_1)^2]^{0{,}5} = 0{,}35830039 \cdot L_{Q1}$ and $R_1 = 0{,}5 \cdot D_1$

$g_2(x) = 0{,}5 * D_2 + D_2 / L_{Q1} \cdot x$

[Originally one has the line-function "$g_2(D,x) = 0{,}5*D + D/L_{Q1}*x$" and besides the x-value also the diameter $D = D_2$ is wanted by the equation "$g_1(x) = g_2(x)$". The wanted x-value "$x = c_{R2L2}$" and diameter D_2 interfer each other referring to the equation "$g_1(x) = g_2(x)$".]

The coordinate axes are located in the mid of the lower square line. The x-arrow shows to the right and the y-arrow shows upwards.

According to the book [1] on page 93 the measured average value for R_2 is **0,6084409**, which is very near to the value **0,60816347..** by my calculation of R_2 using the square length $L_{Q1} = 1$. The relative deviation is only about 0,05%.

On pages 2 and 3 one can see the more detailed calculation to get the diameter D_2 at the drawing of Leonardo da Vinci. The calculated or graphical solution is presented at sketch 2. For both solution ways one has to choose a starting value named D_{2_0} to get the wanted circle diameter D_2.

I do not know if calculations of this kind were performed by another person or by a group before. This solution approach is really not so difficult and therefore the solution is able to be found also by a spare time mathematician like me.

Yours sincerly,

spare time mathematician from germany

Description of the determination of the diameter D_2:

The diameter D_2 – the circle diameter at the drawing of Leonardo da Vinci - or radius R_2 respectively is unknown at the beginning.
Circle C_1 with radius R_1 is equal in surface to square Q_1 with length L_{Q1} according to book [1].
Circle C_2 with radius R_2 is equal in surface to square Q_2 with length L_{Q2} according to book [1].

There are four points of intersection between the two circles (C_1 and C_2) and the upper lines of the two squares (Q_1 and Q_2), each located to the right and to the left of the vertical symmetry axis at x=0. The dimensions of the four intersections to the vertical symmetry axis are (see also sketch 2):

$c_{R1L1} = [R_1^2 – (L_{Q1}-R_1)^2]^{0,5}$ --> the intersection S_{R1L1} of the upper line of square Q_1 and circle C_1
$c_{R1L2} = [R_1^2 – (L_{Q2}-R_1)^2]^{0,5}$ --> the intersection S_{R1L2} of the upper line of square Q_2 and circle C_1
$c_{R2L1} = [R_2^2 – (L_{Q1}-R_2)^2]^{0,5}$ --> the intersection S_{R2L1} of the upper line of square Q_1 and circle C_2
$c_{R2L2} = [R_2^2 – (L_{Q2}-R_2)^2]^{0,5}$ --> the intersection S_{R2L2} of the upper line of square Q_2 and circle C_2

Further dimensions are (see calculation at sketch 2):
$\tan(\eta) = L_{Q1} / c_{R1L1}$ which leads to an angle η: $\eta = 70,28738°$..; further: $\tan(\eta) = L_{Q2} / c_{R2L2}$
$\tan(\zeta) = R_2 / (0,5 \cdot L_{Q1})$ the correct value of angle ζ is dependent on the the wanted radius R_2.
The formula for the dimension c_{R2L2} can also be given by $c_{R2L2} = L_{Q2}/\tan(\zeta) – 0,5 \cdot L_{Q1}$

Wanted is the x-value "**$x=c_{R2L2}$**", at which following condition $B_{1\text{-}2}$ is fulfilled.
condition $B_{1\text{-}2}$: the intersection "circle C_2 with the upper line of square Q_2" is located at straight line $g_2(x)$. Also mid point M_2 of circle C_2 and the left, lower edge of square Q_1 are located at $g_2(x)$.

The wanted diameter D_2 and the wanted x-value c_{R2L2} are dependent by each other. There are two unknown terms - namely D_2 and c_{R2L2} -, to whom only one equation – namely $g_1(x) = g_2(x)$ - is opposite. For the solution finding one makes use of an starting value D_{2_0} and of the appropriate straight line function $g_{2\_0}(x)$ to get the exact solution for the x-value c_{R2L2} and for diameter D_2.

In the following and also at sketch 2, the length L_{Q1} of square Q_1 is set: $\mathbf{L_{Q1} = 10}$
One can choose any starting value D_{2_0}, as long as the terms of the roots of the concerning functions take positive values. The starting value $\mathbf{D_{2\_0} = 14}$ is chosen for the following calculation:

Starting value $D_{2\_0} = 14$ ---> $L_{Q2\_0} = D_{2\_0} \cdot \sqrt{(\pi/4)} = 12,407177..$ with
$g_{2\_0}(x) = 0,5 \cdot D_{2\_0} + D_{2\_0} / L_{Q1} \cdot x$ [starting values marked with _0]

Starting value D_{2_0} gives not the correct solution "$x=c_{R2L2}$" using the equation "$g_1(x) = g_{2\_0}(x)$" (see sketch 2), because the condition $B_{1\text{-}2}$ is not fulfilled. You can see that on sketch 2, where the intersection "upper line of square Q_{2_0} with circle C_{2_0}" is not located on straight line $g_{2\_0}(x)$.

This is verified by following calculation with the starting value D_{2_0}:

$g_1(x) = L_{Q1} / c_{R1L1} \cdot x = g_{2\_0}(x) = 0,5 \cdot D_{2\_0} + D_{2\_0} / L_{Q1} \cdot x$ -->
$x = c_{R2L2\_0} = 0,5 \cdot D_{2\_0} / (L_{Q1}/c_{R1L1} - D_{2\_0}/L_{Q1}) = 0,5 \cdot 14 / (10/3,5830039 – 14/10) = 5,032516..$
$g_1(c_{R2L2\_0}) = L_{Q1} / c_{R1L1} \cdot c_{R2L2\_0} = 10 / 3,5830039.. \cdot 5,032516.. = 14,045523..$
$g_2(c_{R2L2\_0}) = 0,5 \cdot D_{2\_0} + D_{2\_0} / L_{Q1} \cdot c_{R2L2\_0} = 0,5 \cdot 14 + 14/10 \cdot 5,032516.. = 14,045523..$

Referring to the angle η the formula is given: $\tan(\eta) = L_{Q2\_0} / c_{R2L2\_0}^*$ -->
$c_{R2L2\_0}^* = L_{Q2\_0} / \tan(\eta) = 12,407177.. / \tan(70,28738°) = 4,4455.. \neq c_{R2L2\_0} = 5,032516..$

At the x-value c_{R2L2_0} the straight line function $g_1(x)$ crosses the straight line function $g_{2\_0}(x)$, whereas at the x-value $c_{R2L2\_0}^*$ the intersection "upper line of square Q_{2_0} with circle C_{2_0}" is located.

Solution: the intersection of the straight line "$g_{2\_0}(x) = 0,5 \cdot D_{2\_0} + D_{2\_0} / L_{Q1} \cdot x$" with the upper line of square L_{Q2_0} is decisive, the intersection gives the wanted value "$x=c_{R2L2}$", which corresponds to the wanted diameter D_2. The vertical vector (red coloured) starting from this intersection intersects the straight line function $g_1(x)$ at this x-value (see sketch 2). The value of straight line $g_1(x=c_{R2L2})$ is equivalent to the square length L_{Q2}, from which the diameter D_2 and the radius R_2 can be derived. The calculation with the starting value D_{2_0} can be taken from sketch 2 at page 6 of this report.

Only the formula for the wanted x-value c_{R2L2} is presented at this passage (see more at sketch 2):
$c_{R2L2} = L_{Q2\_0}/\tan(\zeta_0) – 0,5 \cdot L_{Q1} = 12,407177.. / 1,4 – 5 = 3,862269..$ with $\tan(\zeta_0) = R_{2\_0}/(0,5 \cdot L_{Q1})$

I do not know, why the solution of the x-value "$x=c_{R2L2}$" underlies the equation "$g_{2\_0}(c_{R2L2})=L_{Q2\_0}$". But this method works. Profi-mathematicians will surely deliver a proof for that.

Proof of the correctness of the calculated values by choosing an optimal starting value:

1) the square length L_{Q1} is set as before to the value **10**.
   Thereby the diameter D_1 results to $D_1 = 11{,}283792..$. from $D_1 = L_{Q1} \cdot \sqrt{(4/\pi)}$

2) a diameter D_{2_0} serves as a starting value. For simplicity D_{2_0} is set to the value of L_{Q1}: $D_{2\_0}=L_{Q1}$
   With this starting value one gets for the angle ζ_0 the formulas $\zeta_0 = \arctan(D_{2\_0} / L_{Q1}) = 45°$ and $\tan(D_{2\_0} / L_{Q1}) = 1$, which simplifies the following calculation immensely.

3) straight line $g_1(x)$ is: $g_1(x) = L_{Q1} / c_{R1L1} \cdot x$ with $c_{R1L1} = [R_1^2 - (L_{Q1}-R_1)^2]^{0,5}$ and $R_1 = 0{,}5 \cdot D_1$

4) straight line $g_2(x)$ is: $g_2(x) = 0{,}5 \cdot D_2 + D_2 / L_{Q1} \cdot x$
   starting straight line $g_{2\_0}(x)$ is: $g_{2\_0}(x) = 0{,}5 \cdot D_{2\_0} + D_{2\_0} / L_{Q1} \cdot x = 0{,}5 \cdot L_{Q1} + x$ with $D_{2\_0}=L_{Q1}$

$L_{Q2\_0} = D_{2\_0} \cdot \sqrt{(\pi/4)} = L_{Q1} \cdot \sqrt{(\pi/4)} = 8{,}8622693..$ with $L_{Q1} = 10$;

$c_{R2L2} = L_{Q2\_0} / \tan(\zeta_0) - 0{,}5 \cdot L_{Q1} = L_{Q1} \cdot \sqrt{(\pi/4)} - 0{,}5 \cdot L_{Q1} = L_{Q1} \cdot [\sqrt{(\pi/4)} - 0{,}5] = 3{,}8622693..$

$c_{R1L1} = [R_1^2 - (L_{Q1}-R_1)^2]^{0,5} = L_{Q1} \cdot [\sqrt{(4/\pi)} - 1]^{0,5} = 3{,}5830039..$

$L_{Q2} = c_{R2L2} \cdot \tan(\eta) = c_{R2L2} \cdot L_{Q1} / c_{R1L1} = L_{Q1} \cdot [\sqrt{(\pi/4)} - 0{,}5] / [\sqrt{(4/\pi)} - 1]^{0,5} = 10{,}7794169..$

$D_2 = L_{Q2} \cdot \sqrt{(4/\pi)} = L_{Q1} \cdot [1 - 0{,}5 \cdot \sqrt{(4/\pi)}] / [\sqrt{(4/\pi)} - 1]^{0,5} = 12{,}1632695..$

$R_2 = 0{,}5 \cdot D_2 = 0{,}5 \cdot L_{Q1} \cdot [1 - 0{,}5 * \sqrt{(4/\pi)}] / [\sqrt{(4/\pi)} - 1]^{0,5} = 6{,}0816347..$

Proof: $g_2(c_{R2L2}) = 0{,}5 \cdot D_2 + D_2 \cdot c_{R2L2} / L_{Q1} = 6{,}0816347.. + 12{,}1632695.. \cdot 3{,}8622693.. / 10$
---> $g_2(c_{R2L2}) = 10{,}7794169..$ $[= g_1(c_{R2L2}) = L_{Q1} / c_{R1L1} \cdot c_{R2L2} = 10 / 3{,}5830039 \cdot 3{,}8622693]$

The formulas show, that the values of diameter and square lengths are not dependent on the golden section. But perhaps there is still something hided in the drawing of Leonardo da Vinci, which at the present time is not seen or perceived. In this context not only the golden section is mentioned.

The calculated value of radius R_2 is: $R_2 = 6{,}0816347..$ for given value $L_{Q1} = 10$.
The calculated arm radius R_φ derived from R_2 (see sketch 2, solution) is : $R_\varphi = 4{,}5355941..$.

The measure value of the arm radius R_φ at the drawing of Leonardo da Vinci is according to the book [1] (on page 97 the arm radius is named "x" and related to a square length $L_{Q1}=1$) $R_\varphi = 0{,}436$.
Here the measured arm radius is set to $R_\varphi = 4{,}36$ because of the square length $L_{Q1}=10$.
Notice: in the radius range of the solution value R_2 (= 6,0816347) small changes of radius R_2 cause relative big changes of the arm radius R_φ.

The leg position of the proportion study could be comprehend with a line named line l_B.
Line l_B goes through the mid point M_2 of the circle C_2 and through the upper intersection point of circle C_1 with square Q_2. The angle ξ between the vertical and line l_B **is 26,397°** (see sketch 2).
At page 72 of the book [1] the quotation is given: "..., that the centre of the outermost points of the outstretched extremities is the navel and the space between the legs is an equilateral triangle."
[Translation by the author may be not correct]
In this context it would be helpful to know the measured distance of this equilateral triangle to the bottom line (at y=0). This could give a better understanding of the leg position in the drawing of Leonardo da Vinci.

By that all four points of intersections between the 2 circles and the 2 squares – given by the four dimensions c_{R1L1}, c_{R2L1}, c_{R1L2}, c_{R2L2} – would be integrated in the drawing of Leonardo da Vinci.
- c_{R1L1} (intersection S_{R1L1}) yields the straight line $g_1(x)$
- c_{R2L2} (intersection S_{R2L2}) yields the straight line $g_2(x)$ and therefore the radius R_2
- c_{R2L1} (intersection S_{R2L1}) yields the arm radius R_φ
- c_{R1L2} (intersection S_{R1L2}) yields the straight line l_B - and by that eventually the leg position?

Conclusion:

Isn't it astonishing, how the measures of the human body fit into the square and the circle of the drawing of Leonardo da Vinci?
And isn't it a little bit strange, how this can be simply proved by mathematical formulas?

One should lead to mind (independent wether, if one is convinced that Leonardo da Vinci applied this presented method or not):
with application of the presented method and with help of the formula "$D=L\cdot\sqrt{(4/\pi)}$" – which means surface equality for square/circle – for the two square/circle-pairs, there is only one definite solution for the diameter D_2. And the square Q_1 and the circle C_2 with this calculated diameter D_2 fit perfectly to the Vitruvian man (the standard man) as presented by Leonardo da Vinci at his drawing.

With a diameter D_{2_GS} for example, which depends on the Golden Section ($D_{2\_GS} = 2 \cdot 0{,}618034 = 1{,}236068$ related to a square length $L_{Q1} = 1$), this perfectness is not possible.
The diameter D_{2_GS} does not allow an arm movement of the human body as drafted at the proportion study of Leonardo da Vinci.

With the constellation of the geometries as given at the proportion study, a circle C with a diameter $D = 1{,}25$ related to a square length $L_Q = 1$ goes through the upper edges of this square.
And the diameter $D_{2\_GS} = 1{,}236068$ is pretty narrow to the value $D = 1{,}25$.
Keep also the statement at page 3 in mind: small changes of the calculated diameter D_2 (related to square length $L_{Q1}=1$, diameter $D_2 = 1{,}216327$) leads to relative big changes of the arm radius R_φ!

Sketch 3 shows the method with **equality of circumference** referring to circle and square.
The calculated value of radius R_2 is: $\mathbf{R_2 = 3{,}4758379..}$ for given value $\mathbf{L_{Q1} = 10}$.
Here one has a completely different constellation of the geometries compared to those, which are given by the surface equality of square and circle.

The method described in this report, which underlies the quadrature of the circle, delivers a plausible and pretty simple solution for the diameter at the drawing of Leonardo da Vinci. This is supported by the very small deviation of the calculated radius R_2 to the measure value according to book [1].
By that one can conclude, that Leonardo da Vinci reproduced very exactly the geometries - two times the quadrature of circle referring to surface equality – and unified it with the Vitruvian man at his drawing.

Questions referring to the measures of the proportion study:

The diameter at da Vinci´s drawing of the proportion study is **220 mm** according to a report from 2015 of Vitor Murtinho. The square length is measured to 181,5 mm according to this report.

My question especially to art scientists is: 500 years ago, in this era, was the metric system or a length system similar to it already known?

Since the antique era the approximation **"22 : 7"** has been known for the circular figure π.
Can it be that there is a connection between the measure 220 mm of the diameter at da Vinci´s drawing and the figure 22 in the approximation formula for the circular figure?

Leonardo da Vinci's proportion study: method (new?) inspired by the book of K. Schröer / K. Irle.
Wanted is the intersection of straight lines $g_1(x)$ with $g_2(x)$ at equality in surface of square and circle.
$g_1(x)$ and $g_2(x)$ cross each other at the intersection of upper line of square Q_2 with circle C_2. The values of L_{Q2} and D_2 are solvable graphically and analytically.

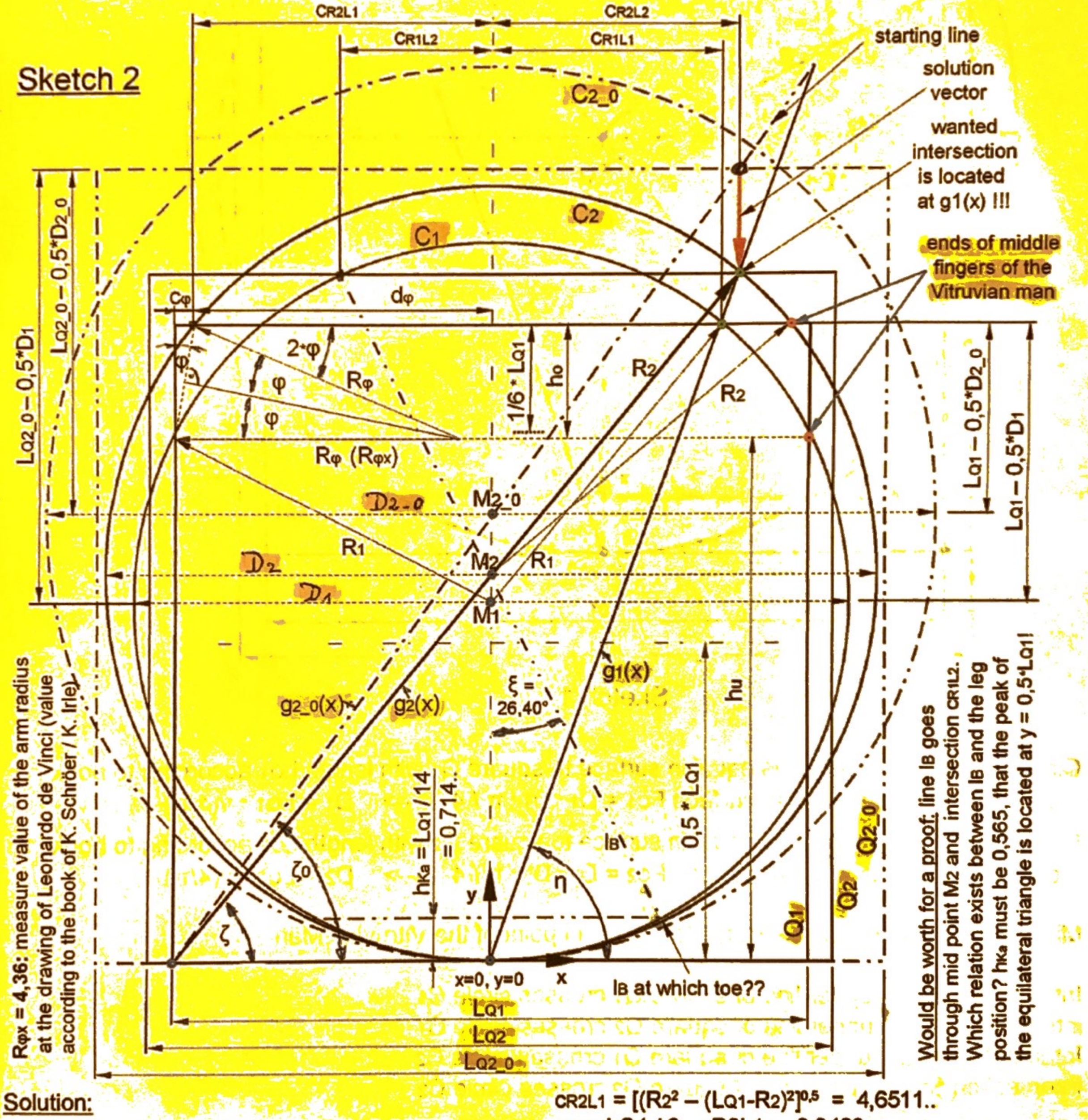
Square length $L_{Q1} = 10$
Circle-Ø $D_1 = L_{Q1} \cdot \sqrt{(4/\pi)} = 11{,}283792..$
from "$L_{Q1} \cdot L_{Q1} = D_1 \cdot D_1 \cdot \pi / 4$" (surface equality)

Circle-Ø $D_{2\_0} = 14$ (Starting value marked with _0)
Square length $L_{Q2\_0} = D_{2\_0} \cdot \sqrt{(\pi/4)} = 12{,}407177..$

Measure: 1 is equivalent to 1 cm

$R_1 = 0{,}5 \cdot D_1$ and $R_2 = 0{,}5 \cdot D_2$
$g_1(x) = L_{Q1} / c_{R1L1} \cdot x$ with
$c_{R1L1} = [R_1^2 - (L_{Q1}-R_1)^2]^{0,5} = 3.5830..$
$g_2(x) = 0{,}5 \cdot D_2 + D_2 / L_{Q1} \cdot x$
$\tan(\eta) = L_{Q1} / c_{R1L1}$
$\eta = 70{,}2874°..$
$\tan(\zeta_0) = R_{2\_0} / (0{,}5 \cdot L_{Q1})$
$\zeta_0 = 54{,}4623°..$
$h_u = R_1 + (R_1^2 - 0{,}25 \cdot L_{Q1}^2)^{0,5} = 8.2555..;$
$h_o = L_{Q1} - h_u = 1{,}7445..$

Sketch 2

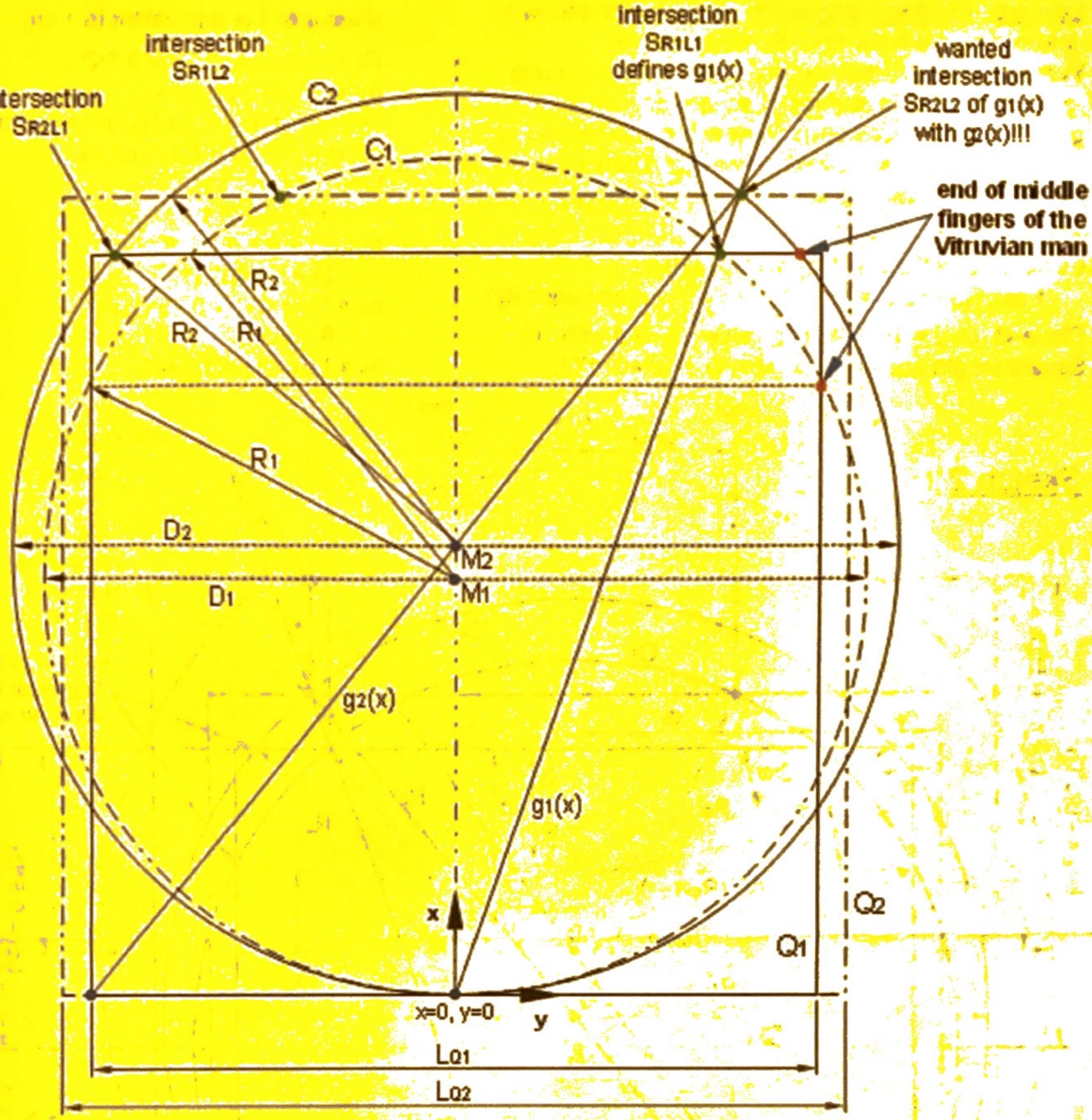


Solution:

$c_{R2L2} = L_{Q2\_0} / \tan(\zeta_0) - 0{,}5 \cdot L_{Q1} = 3{,}86227..$
$L_{Q2} = c_{R2L2} \cdot \tan(\eta) = 10{,}77942..$
$D_2 = L_{Q2} \cdot \sqrt{(4/\pi)} = 12{,}16327..$
$R_2 = D_2 / 2 = 6{,}081635..$
$\zeta = \arctan[L_{Q2} / (0{,}5 \cdot L_{Q1} + c_{R2L2})] = 50{,}57476°..$
$g_1(x{=}c_{R2L2}) = L_{Q1} / c_{R1L1} \cdot x = 10{,}77942..$
$g_2(x{=}c_{R2L2}) = 0{,}5 \cdot D_2 + D_2 / L_{Q1} \cdot x = 10{,}77942..$

$c_{R2L1} = [(R_2^2 - (L_{Q1}-R_2)^2]^{0,5} = 4{,}6511..$
$c\varphi = LQ1 / 2 - cR2L1 = 0{,}3489..$
$\varphi = \arctan(c\varphi / h_o) = 11{,}3101°..$
$R\varphi = h_o / \sin(2*\varphi) = 4{,}5356..$

Measure value: $\mathbf{R\varphi x = 4{,}36}$; $\varphi x = \arcsin(h_o/R\varphi x) / 2$
$\varphi x = 11{,}7926°..;$ $c\varphi x = h_o * \tan(\varphi x) = 0{,}3642..;$
$d\varphi x = L_{Q1} / 2 - c\varphi x = 4{,}6358..;$
$R2\varphi x = (L_{Q1}^2 + d\varphi x^2) / (2*L_{Q1}) = 6{,}0745..$



Sketch 1

Circle C_1 with diameter D_1 is equal in surface to square Q_1 with length L_{Q1} according to book [1]:

surface $F_{Q1} = L_{Q1} \cdot L_{Q1}$ = surface $F_{C1} = D_1 \cdot D_1 \cdot \pi / 4$ ---> $D_1 = L_{Q1} \cdot \sqrt{(4/\pi)}$

Circle C_2 with diameter D_2 is equal in surface to square Q_2 with length L_{Q2} according to book [1]:

surface $F_{Q2} = L_{Q2} \cdot L_{Q2}$ = surface $F_{C2} = D_2 \cdot D_2 \cdot \pi / 4$ ---> $D_2 = L_{Q2} \cdot \sqrt{(4/\pi)}$

Mid point M_2 of circle C_2 coincides to the navel point of the Vitruvian Man.

Intersection S_{R1L1}: upper line of square Q_1 crosses circle C_1
intersection S_{R1L2}: upper line of square Q_2 crosses circle C_1
Intersection S_{R2L1}: upper line of square Q_1 crosses circle C_2
Intersection S_{R2L2}: upper line of square Q_2 crosses circle C_2

Straight line $g_1(x)$ goes through point [x=0; y=0], through intersection S_{R1L1} and intersection S_{R2L2}

Straight line $g_2(x)$ goes through left, lower edge of square Q_1, through mid point M_2 of circle C_2 and through intersection S_{R2L2}.

The dotted circle C_1 and the dotted square Q_2 at sketch 1 are not represented at the drawing of Leonardo da Vinci.

quadrature of circle due to circumferences:
Wanted is intersection of line $g_1(x)$ with line $g_2(x)$ at circumference equality of square and circle. $g_1(x)$ and $g_2(x)$ cross each other at the intersection of upper line of square Q_2 with circle C_2. The values of L_{Q2} and D_2 are solvable graphically and analytically.

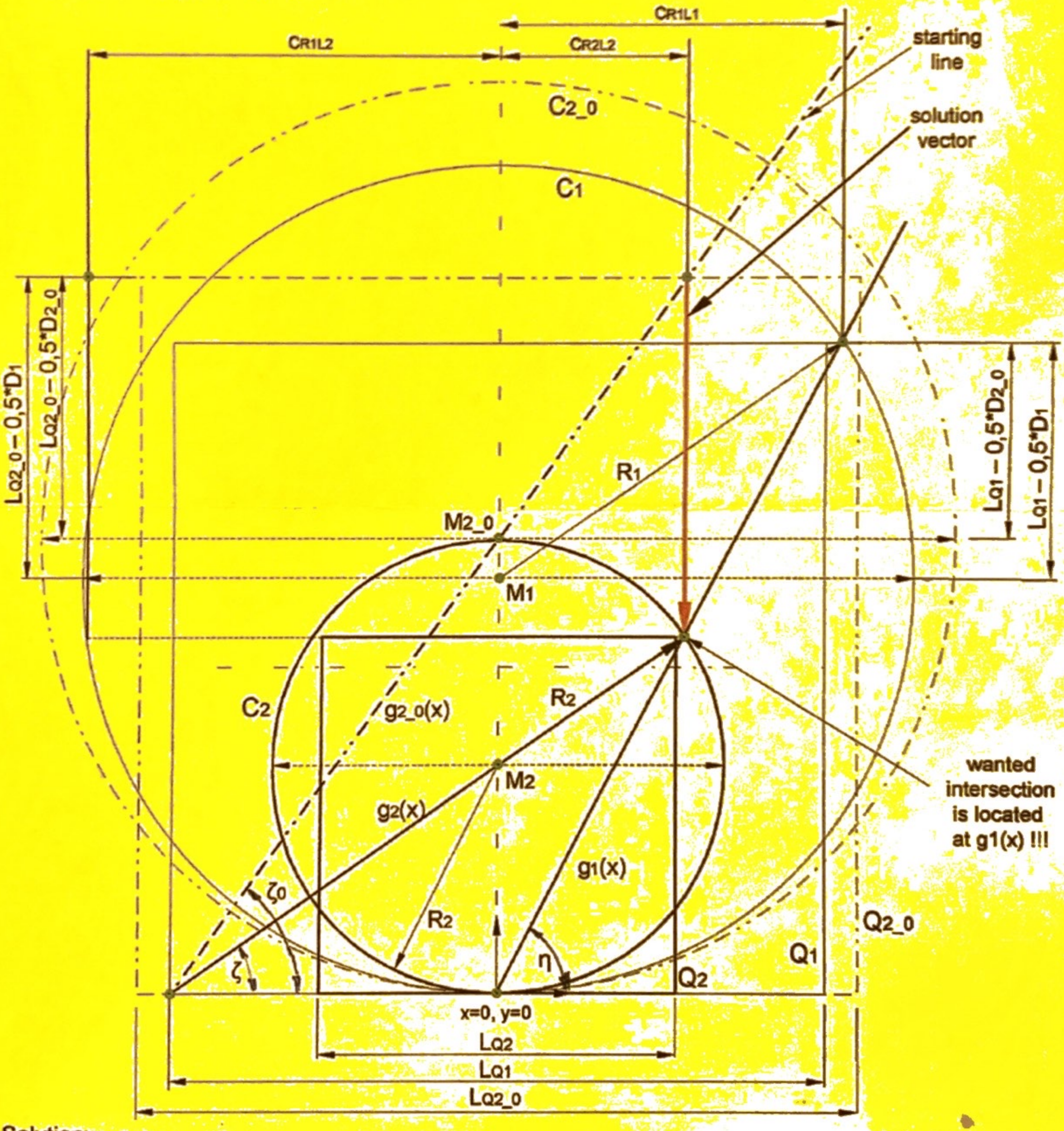
Square length $L_{Q1} = 10$
circle-Ø $D_1 = L_{Q1} \cdot 4 / \pi = 12{,}7324..$
from "$4 \cdot L_{Q1} = D_1 * \pi$"

circle-Ø $D_{2\_0} = 14$ (Starting value marked with "_0")
square length $L_{Q2\_0} = D_{2\_0} \cdot \pi / 4 = 10{,}9956..$

Measure: 1 is equivalent to 1 cm

$R_1 = 0{,}5 \cdot D_1$ and $R_2 = 0{,}5 \cdot D_2$
$g_1(x) = L_{Q1} / c_{R1L1} \cdot x$ mit
$c_{R1L1} = [R_1^2 - (L_{Q1} - R_1)^2]^{0,5} = 5.2272..$
$g_2(x) = 0{,}5 \cdot D_2 + D_2 / L_{Q1} \cdot x$
$\tan(\eta) = L_{Q1} / c_{R1L1}$
$\eta = 62{,}4029°..$
$\tan(\zeta_0) = R_{2\_0} / (0{,}5 \cdot L_{Q1})$
$\zeta_0 = 54{,}4623°..$

Sketch 3



Solution:
$c_{R2L2} = L_{Q2\_0} / \tan(\zeta_0) - 0{,}5 * L_{Q1} = 2{,}8540..$
$L_{Q2} = c_{R2L2} \cdot \tan(\eta) = 5{,}4598..$
$D_2 = L_{Q2} \cdot 4 / \pi = 6{,}9517..$
$R_2 = D_2 / 2 = 3{,}4758..$
$\zeta = \arctan[L_{Q2} / (0{,}5 \cdot L_{Q1} + c_{R2L2})] = 34{,}8058°..$
$g1(x = c_{R2L2}) = L_{Q1} / c_{R1L1} * x = 5{,}4598..$
$g2(x = c_{R2L2}) = 0{,}5 * D_2 + D_2 / L_{Q1} * x = 5{,}4598..$

surfaces and circumferences:
$Fl_{Q1} = L_{Q1}^2 = 100$
$Fl_{R1} = R_1^2 \cdot \pi = 127{,}324..$
$U_{R1} = U_{Q1} = D_1 \cdot \pi = 40$

$Fl_{Q2} = L_{Q2}^2 = 29{,}810..$
$Fl_{R2} = R_2^2 \cdot \pi = 37{,}955..$
$U_{R2} = U_{Q2} = D_2 \cdot \pi = 21{,}839..$